

Minnesota State High School Mathematics League Individual Event

2004-05 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Find a point (x, y) that lies on the graphs of both $\frac{5}{3}x - \frac{1}{2}y = 4$ and $-4x + 3y = -6$.

- k = _____ 2. Choose k so that the system below has a solution when $x = 1$.

$$2x - 4y = 6$$

$$2x + y = k$$

3. There are many solutions to the set of three equations below. Express y and z in terms of x so that every real value of x gives a solution to these equations:

y = _____
z = _____

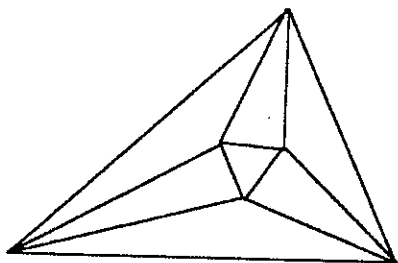
$$2x - y + z = 8$$

$$3x - y + 2z = 11$$

$$5x - y + 4z = 17$$

- _____ 4. Five students work to cover a wire frame with crepe paper flowers to make a float for a parade. Each student can make 3 flowers a minute. Alicia and Beth have experience attaching the flowers to the frame, and each can attach 5 flowers per minute. It takes 3000 flowers to cover the frame. If the two girls help the others make flowers at the beginning of the evening, and then switch to attaching them while the others continue to make flowers, how long (minutes) should they make flowers before they begin to attach flowers to the frame so that the group finishes the job in the minimum time?

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2004-05 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Four equilateral triangles of side length s are arranged as in Figure 1 to form a parallelogram. In terms of s , what is the length of its short diagonal?
- _____ 2. $\triangle ACD$ is inscribed in a regular pentagon $ABCDE$. What, in degrees, is the measure of $\angle ACD$?
- _____ 3. Four equilateral triangles of side length s are arranged as in Figure 1 to form a parallelogram. In terms of s , what is the length of its long diagonal?
- _____ 4. The isosceles trapezoid $ABCD$ inscribed in a semicircle (Figure 4) has sides of length 6 and $BC = 14$. How long is the diameter of the semicircle?

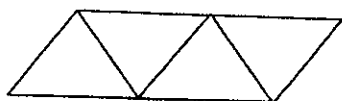


Figure 1

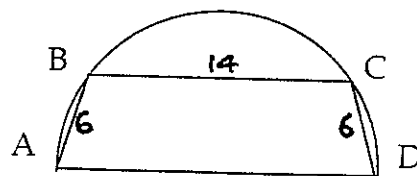
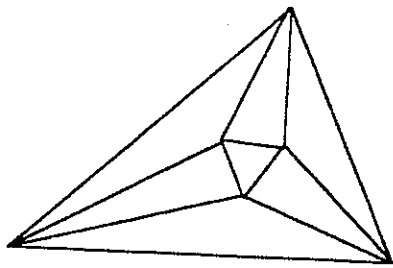


Figure 4

Name _____ Team _____



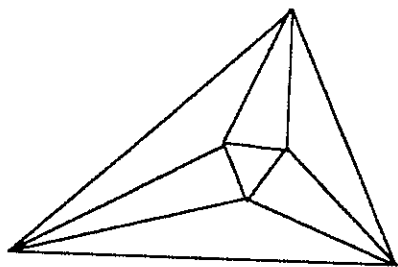
Minnesota State High School Mathematics League Individual Event

2004-05 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Give both coordinates of the lowest point on the graph of $y = \sin^{-1} x$ (sometimes written as $y = \text{Arcsin } x$). Angles are expressed in radians.
- _____ 2. Describe the set of all solutions to $\sin^2 x - 4\sin x + 4 = 0$.
- _____ 3. Express the square roots of $-i$ where $i = \sqrt{-1}$ in the form $a + bi$ where a and b are rationalized fractions.
- _____ 4. Given that the sides of a regular pentagon $ABCDE$ all have length 1, express the length of diagonal AC in the form $a(\cos b^\circ)$ where a and b are integers.

Name _____ Team _____



Minnesota State High School Mathematics League

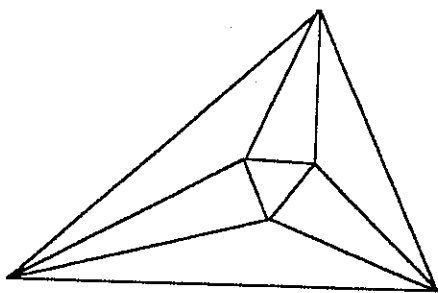
Individual Event

2004-05 Event 3D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Express $[4^{-1} + 9^{-1} + 36^{-1}]^{-1}$ in decimal form.
- _____ 2. Rationalize and simplify $\frac{\sqrt{240} + \sqrt{216}}{\sqrt{60} + \sqrt{54}}$.
- _____ 3. If $\log_8 A = r$, what (in terms of r) is $\log_2 \sqrt{A} + \log_4 \sqrt{A}$?
- _____ 4. Given that $0 < M < 1 < N$ and that $\log_M N = \log_N M$, what is $\log_2 MN$?

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2004-05 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

- _____ 1. Find a point (x, y) such that for every real choice of k ,

$$(2x - 3y + 1) + k(-5x + 4y + 8) = 0$$
- _____ 2. In Team Event 2, we found that if a point P is placed inside a rectangle $ABCD$ so that $AP = \sqrt{13}$, $CP = \sqrt{117}$, and $DP = \sqrt{45}$, then $BP = \sqrt{85}$. If the sides of this rectangle are integers, what must be its perimeter?
- _____ 3. Bhaskara, a Hindu mathematician working more than a thousand years ago, posed this problem. A pole casts a shadow of a certain length. Later in the day, it is noted that the shadow has increased 19, and the distance from the top of the pole to the end of the shadow has increased by 13. He must have had integer answers in mind, but I wonder if he knew that even then there is more than one solution. Find the two smallest integer values of the lengths of the shadow, assuming that we require shadows to have positive lengths.
- _____ 4. $\triangle ABC$ has sides of length $AB = 7$, $AC = 5$, and $BC = 10$. Its medians intersect at M . How long is AM ?
- _____ 5. Accurate to three places to the right of the decimal, give the radian measure of all angles between 0 and 6.283 that satisfy $6\sin^2 \theta = \cos \theta + 4$.
- _____ 6. If $x > 0$, $x \neq 1$, $y > 0$, $y \neq 1$, and $y^{\log_x y^2} = \sqrt[3]{x^{20} y^7}$, then either $y = f(x)$ or $y = g(x)$ where f and g are both exponential functions of x . Find them.
 $f(x) =$ _____
 $g(x) =$ _____

Team _____